Quantum inertial force and its consequences

Article · September 2015

1 author:

Arbab I. Arbab
Qassim University

Quantum inertial force and its consequences

A. I. Arbab [(a)]

Department of Physics, Faculty of Science, University of Khartoum, P.O. Box 321, Khartoum 11115, Sudan

PACS 03.65.Ta – Foundations of quantum mechanics
PACS 03.50.De – Classical electromagnetism, Maxwell equations
PACS 41.20.Jb – Electromagnetic wave propagation
PACS 03.75.-b – Matter waves

Abstract. - Inertial quantum equation of a moving particle is derived from our unified quantum equation. The self-inertial quantum force on a particle of mass m moving with constant velocity $\vec{v}$ is found to be $\vec{F}_m = -\frac{m^2c^2}{\hbar}\,\vec{v}$, where $\hbar$ and c are the Planck's constant and speed of light, respectively. This force is found to manifest the perpetual process of creation/annihilation that a moving particle undergoing. The origin of inertial quantum force is found to have a quantum aspect. When a charged (q) particle moves in a magnetic field with constant velocity, the critical magnetic field that makes the charge and mass move concurrently is $B_{cr} = \frac{m^2c^2}{q\hbar}$. In gravity, the angular momentum of the particle moving with constant velocity at a distance r from another particle is given by $L = \frac{Gm\hbar}{c^2r}$. A spinning particle in gravity whose radius is equal to Schwarzschild radius has a spin equal to $S = \hbar/2$, and that with radius equals to the classical electron radius will have spin $S = \hbar$. As in Unruh effect, where an accelerating observer sees thermal radiation with temperature $T = \frac{\hbar a}{2\pi c k_B}$, a particle moving with speed v is found to exhibit same effect if $a = \frac{mc^2}{\hbar}\,v$, where k_B is the Boltzmann's constant. The radiation is found to extend only for a distance equals to the Compton wavelength. When the moving charged particle (magnetic moment) is placed in an external magnetic field, the particle precesses with Larmor frequency.

Introduction. – According to Newton's second law the inertia is defined as the resistance of an object to change its state. A particle at rest will experience a resistance when accelerated, and a particle moving at constant speed will resist when it is decelerated. Newton's quantify this in his second law as an inertial force will occur whenever the object changes its velocity. The inertia is also found to be a property of the object in question. It is an inherent property of matter. It can be related to the material content of the object. This is referred to the mass of the object. Mach however hypothesized that inertia is not a mere property of the object, but reflects how much matter exists around the object [1]. Hence, in free space inertia is zero. However, the inertia doesn't occur unless the object undergoes a change in state of motion. The cause of this change can be gravitational, electromagnetic, etc. Whatever the type of external force, it is always equal to ma. In quantum mechanics, it is difficult to observe the velocity change of a particle as the particle is deemed not to have a definite trajectory. This difficulty makes the inertial force ambiguous. One may also anticipate that inertia could be a pure quantum effect.

De Broglie hypothesized that a micropparticle is endowed with wave nature [2]. This is however unlike the wave

(a) aiarbab@uofk.edu : arbab.ibrahim@gmail.com: Tel:+249122174858

of light which results from the oscillations of the electromagnetic field. It is a material wave that can be envisaged as a wavepacket rather than a pure wave. This wave is concomitant with the particle motion. *It is like an object and its shadow that moving with same velocity.* Light, as an electromagnetic wave, exhibits a particle nature as observed in photoelectric and Compton effects. Hence particle and waves have dual behavior. It can exhibit a given behavior whenever the necessary conditions are met to make it observable. This duality can also be seen in Einstein mass-energy relation ($E = mc^2$) where energy is wave and mass is matter. The analogy between waves and matter, however, remains to be incomplete. Though the particle has inertia, no inertia is associated with matter waves in the formulation of quantum mechanics. This asymmetry is not exhibited in the quantum formulation of matter waves.

One of our concerns here is to accommodate this missing part in a unified and symmetric picture of quantum and wave mechanics. We provide in this paper a new formulation of quantum mechanics that reflects this property, and the phenomena associated with it. Since Newton's second law is not appropriate to be applied to a quantum particle, we have found the new formulation provided an analogue of a quantum Newton's law that is applicable to quantum particles. Owing to this formulation, the inertia of the wave is reflected in the wave equation describing the matter wave (particle). This term appears like a velocity force (drag/viscous) in quantum Newton's law. While inertial force in Newton's law relies on the change of velocity, quantum inertial force depends on the velocity of the particle which can be easily determined. We can say that we have found a quantum inertial force. This force is always present whether the particle accelerates or not. It is related to the process of creation and annihilation that a moving quantum particle undergoes even when its speed is constant. The application of this force to the motion of a particle in gravitational and electromagnetic fields leads to consistent results that were obtained from quantum electrodynamics calculations. This agreement encourages us to further consider the application of this formulation to study other issues in atomic physics and optics.

Unified quantum mechanics. – A quaternionic formation of quantum mechanics yields a system of equations expressed as [3,6]

$$\vec{\nabla}\cdot\vec{\psi}-\frac{1}{c^2}\frac{\partial\psi_0}{\partial t}-\frac{m}{\hbar}\,\psi_0=0\,, \tag{1}$$

$$\vec{\nabla}\psi_0-\frac{\partial\vec{\psi}}{\partial t}-\frac{mc^2}{\hbar}\,\vec{\psi}=0\,, \tag{2}$$

and

$$\vec{\nabla}\times\vec{\psi}=0\,. \tag{3}$$

These equations can be solved to give [3,6]

$$\frac{1}{c^2}\frac{\partial^2\vec{\psi}}{\partial t^2}-\nabla^2\vec{\psi}+\frac{2m}{\hbar}\frac{\partial\vec{\psi}}{\partial t}+\frac{m^2c^2}{\hbar^2}\,\vec{\psi}=0\,, \tag{4}$$

and

$$\frac{1}{c^2}\frac{\partial^2\psi_0}{\partial t^2}-\nabla^2\psi_0+\frac{2m}{\hbar}\frac{\partial\psi_0}{\partial t}+\frac{m^2c^2}{\hbar^2}\,\psi_0=0\,, \tag{5}$$

It interesting that eqs.(4) and (5) are of the generalized Telegraph equation governing the propagation of electric signals in transmission lines [7]. These equations are found to produce Schrodinger, Dirac and Klein-Gordon equations [3]. Equations (4) and (5) embody a resistance (inertia) force because of the presence of the damping term (third term) that acts like a drag force. The general solution of eq.(4) and (5) is of the form

$$\psi_0(r,t) = C\,\exp\left(-mc^2t/\hbar\right)\,\exp\,i(\vec{k}\cdot\vec{r}\pm\,kct)\,, \tag{6}$$

where C is a constant. Despite the damping of the wave amplitude (like a damped harmonic oscillator), the frequency and wavelength of the moving wave (particle) remain invariant keeping the integrity of the particle intact. The dissipative equations in (4) and (5) reminds us with a damped harmonic oscillator that under the action of a force that is proportional to the particle velocity (e.g., a viscous fluid, air resistance, etc.). A particle under damping will very soon move as a free particle when its velocity researches the terminal velocity. This dissipation reflects the interaction of the moving particle with the environment in which it is moving, or from some internal reaction. A quantum dissipative system is well described by Caldeira-Leggett [4, 5].

The inertia of electromagnetic wave. – The electromagnetic field (wave) in free space doesn't experience a resistance (inertia). However, in a conducting medium we expect the resistance force acting like inertia to occur. The electromagnetic fields in a conducting medium satisfy [8]

$$\frac{1}{c^2}\frac{\partial^2\vec{A}}{\partial t^2}-\nabla^2\vec{A}+\vec{\nabla}\left(\vec{\nabla}\cdot\vec{A}+\frac{1}{c^2}\frac{\partial\varphi}{\partial t}\right)-\mu_0\vec{J}=0\,, \tag{7}$$

and

$$\frac{1}{c^2}\frac{\partial^2\varphi}{\partial t^2}-\nabla^2\varphi-\frac{\partial}{\partial t}\left(\vec{\nabla}\cdot\vec{A}+\frac{1}{c^2}\frac{\partial\varphi}{\partial t}\right)-\frac{\rho}{\varepsilon_0}=0\,, \tag{8}$$

where the electric and magnetic fields are defined as

$$\vec{E}=-\vec{\nabla}\varphi-\frac{\partial\vec{A}}{\partial t}\,,\qquad \vec{B}=\vec{\nabla}\times\vec{A}\,. \tag{9}$$

The two potentials are not independent but related by the Lorenz gauge condition (a relativistic gauge)

$$\vec{\nabla}\cdot\vec{A}+\frac{1}{c^2}\frac{\partial\varphi}{\partial t}=0\,. \tag{10}$$

If the third term in the parenthesis in eq.(8) is not zero, then it can be seen as representing resistance (inertia) of the wave propagating in a medium. The pure wave in a vacuum becomes matter wave in a medium and so should be governed by quantum mechanics laws. This can be achieved if we modify the right hand side of eq.(10) to encompass a mass term multiplied by the scalar function (φ), in general, *viz.*,

$$\vec{\nabla}\cdot\vec{A}+\frac{1}{c^2}\frac{\partial\varphi}{\partial t}=\kappa\,m\,\varphi\,, \tag{11}$$

where κ is some constant to adjust the dimension and the necessary sign.

Moreover, using eq.(9) and the fact $\vec{J} = \sigma\,\vec{E}$, in a conducting medium, eq.(7) yields

$$\frac{1}{c^2}\frac{\partial^2 \vec{A}}{\partial t^2} - \nabla^2 \vec{A} + \mu_0\sigma\frac{\partial \vec{A}}{\partial t} + \vec{\nabla}\left(\vec{\nabla}\cdot\vec{A} + \frac{1}{c^2}\frac{\partial \varphi}{\partial t} + \mu_0\sigma\,\varphi\right) = 0\,. \tag{12}$$

If this equation is to satisfy a telegraph-like equation (damped oscillation), then the last term should be set to zero, *i.e.*,

$$\vec{\nabla}\cdot\vec{A} + \frac{1}{c^2}\frac{\partial \varphi}{\partial t} = -\mu_0\sigma\,\varphi\,. \tag{13}$$

Comparing eq.(11) and (13) reveals that $m \propto \sigma$. Moreover, eqs.(4) and (12) yield $\sigma = \frac{2m}{\mu_0\hbar}$. Hence, the electric conductivity has the action of mass (inertia). Therefore, in a conducting medium the Lorenz gauge condition is violated, and consequently gauge invariance is broken. For this reason, we should abandon gauge invariance when we consider the propagation of waves in a medium. It is thus valid only for a wave propagating in vacuum.

The continuity equation - the energy conservation equation. Now multiply eq.(1) by ψ_0 and take the dot product of $\vec{\psi}$ and eq.(2), and add the resulting equations to obtain

$$\frac{\partial u_m}{\partial t} + \vec{\nabla}\cdot\vec{S}_m = -\frac{2m}{\hbar}\,u_m\,, \tag{a}$$

where

$$u_m = \frac{1}{2}\,\left(\psi_0^2 + c^2\psi^2\right)\,, \qquad \vec{S}_m = -\psi_0\,c^2\,\vec{\psi}\,. \tag{b}$$

The above equations state that because of the mass term the system reveals a dissipative behavior. Here u_m and $\vec{S}_m$ describe respectively the energy density and the Poynting vector (energy flow). This equation can be interpreted as energy conservation of a particle described by the fields, ψ_0 and $\vec{\psi}$. It is analogous to the energy conservation of electromagnetic field that is given by

$$\frac{\partial u_{em}}{\partial t} + \vec{\nabla}\cdot\vec{S}_{em} = -\vec{J}\cdot\vec{E}\,, \tag{c}$$

where

$$u_{em} = \frac{1}{2}\,\left(\varepsilon_0 E^2 + \varepsilon_0 c^2\,B^2\right)\,, \qquad \vec{S}_{em} = -\varepsilon_0\vec{B}c^2\times\vec{E}\,. \tag{d}$$

In a conducting medium, one has $\vec{J} = \sigma\,\vec{E}$, where σ is the electric conductivity. Hence, the analogy here applies to the electromagnetic field in a conducting medium. Comparison of the two systems (eq.(a) & (c) and (b) & (d)) reveals that the matter field does't have a magnetic field but some scalar mimicking it, while the matter electric field is taken along the field $\vec{\psi}$. The energy of the matter field flows opposite to the latter matter field. It thus interesting to note that the new formulation of matter and charge fields brings a unified picture of quantum and electromagnetic field theories. The charge and mass are treated as two equivalent analogues. While the electromagnetic energy flows in a direction normal to the electric and magnetic fields, the matter energy flows in a direction parallel to matter electric filed (acceleration).

Formulation of matter wave like electromagnetic fields. – The physical meaning of the vector wavefunction representing a quantum mechanical state, $\vec{\psi}$, can be well understood if we compare Maxwell equations, eqs.(7) and (8) with our quantum equations, eqs.(4) and (5). With an electric charge one can associate electric and magnetic fields. These fields satisfy Maxwell equations. In the same manner, we should look for the analogue fields that are associated with mass (inertia). To keep the same analogy, we call these two fields *inertial electric field* and *inertial magnetic field*, E_m and B_m, respectively. When a mass is placed in these two fields a Lorentz-like force should emerge that is similar to the force acting on a moving electric charge in the presence of electric and magnetic fields. The solution of Maxwell's equations reveals that two fields associated with electric charge satisfy a wave equation. Hence, the electromagnetic fields are waves carrying energy and momentum. With the same token, we have to demonstrate that the two fields associated with moving mass should satisfy the quantum equation we are looking for. In the quantum theory, the matter wave was treated as a longitudinal wave, and no transverse wave associated with particles is thought of. For this purpose, the scalar wavefunction ψ_0 can be associated with a longitudinal wave, whereas a linear combination between ψ_0 and the vector wavefunction $\vec{\psi}$ be associated with the transverse wave for a quantum particle. In the ordinary formulation of quantum mechanics a particle is not described by a vector wavefunction, however. However, in Maxwell formulation the photon is described by the scalar and vector potentials which we will later treat them as the photon wavefunctions.

The inertial electric and magnetic field can, analogously, be defined as

$$\vec{E}_m = -\vec{\nabla}\psi_0 - \frac{\partial\vec{\psi}}{\partial t}\,, \qquad \vec{B}_m = \vec{\nabla}\times\vec{\psi}\,. \tag{14}$$

Equation (1) yields a gauge - like condition [3]

$$\vec{\nabla}\cdot\vec{\psi} + \frac{1}{c^2}\frac{\partial\psi_0}{\partial t} = -\frac{m}{\hbar}\,\psi_0\,, \tag{15}$$

Interestingly, this gauge condition reduces to that of Lorenz when $m = 0$. This case forbids any inertial behavior or information to be carried by massless particles, since $\vec{E}_m = \vec{B}_m = 0$. The latter development of the dynamics (energy conservation, wave pressure, intensity, etc.) will be treated as the way done in Maxwell description.

Applying eq.(14) in eqs.(1) - (3) will yield

$$\vec{E}_m = \frac{mc^2}{\hbar}\,\vec{\psi}\,, \qquad \vec{B}_m = 0\,. \tag{16}$$

Equation (16) suggests that one can choose $\vec{\psi}$ to be the velocity of the moving particle, $\vec{v}$. Therefore,

$$\vec{E}_m = \frac{mc^2}{\hbar}\,\vec{v}\,. \tag{16a}$$

In this case the magnetic matter field will correspond to a vorticity that the particle can produce. It is interesting that the field $\vec{E}_m$ is a quantum field. The Lorentz-like force acting on a particle of mass m' experiencing the two fields, $\vec{E}_m$

[3] changing ψ_0 by $-\psi_0$, and possibly $m \to 2m$ since a photon disintegrates into a pair of particles.

and $\vec{B}_m$, will then take the form

$$\vec{F}_m = m'(\vec{E}_m + \vec{v}\times\vec{B}_m)\,. \tag{17}$$

The force on a mass m_1 due to a mass m_2 moving with velocity $\vec{v}$ is thus

$$\vec{F}_m = \frac{m_1 m_2\, c^2}{\hbar}\,\vec{v}\,, \qquad\qquad \vec{F}_m = \frac{m_1 c}{\lambda_C}\,\vec{v}\,, \tag{18}$$

where $\lambda_C = \hbar/(m_2 c)$ is the Compton wave length of the moving particle. The self - force of a moving particle is given by

$$\vec{F}_m = \frac{m^2 c^2}{\hbar}\,\vec{v}\,. \tag{19}$$

Equations (18) and (19) can be interpreted as representing the force due to creation/annihilation process that a moving particle is undergoing.

Newton's second law states that

$$\vec{F}_m = m\frac{d\vec{v}}{dt} + \frac{dm}{dt}\,\vec{v}\,, \tag{20}$$

where, $\frac{dm}{dt} = -\frac{m}{\tau}$, represents a constant annihilation rate of the particle mass conforming with Einstein energy mass relation, $E = mc^2$. The time $\tau = \frac{\hbar}{mc^2}$ is a characteristic time manifesting the perpetual creation - annihilation process that the particle experiences while moving. It is consistent with Heisenberg uncertainty relation. We see here that the particle creation and annihilation process will lead to acceleration ($\vec{v}/\tau$). Hence, the force on a particle moving at constant speed is

$$\vec{F}_m = -\frac{m}{\tau}\,\vec{v}\,. \tag{21}$$

Owing to Newton's second law, this force is zero. Equation (20) shows that the force exists even when a particle is moving at constant velocity. Hence, because of the second force in eq.(20) the electron is seen to be jolting as predicted by Schrodinger [9, 10]. The force in eq.(18) represents a velocity-dependent force similar to drag (viscous) force. It is a quantum force acting between two masses (matter waves) due to their inertia. It seems that one particle behaves like a fluid, and the other one moves in that fluid so that it experiences a drag force given by eq.(18). This force will be of importance especially for electrically neutral moving particles (*e.g.*, neutron, atom, etc.). It is absent for macroscopic objects, since in that case $\tau\to\infty$. The presence of force acting on a particle with constant velocity can only be understood if one considers the space to have intrinsic curvature. Hence, we may say that a Minkowski (flat) space for a quantum particle is seen to be curved. Therefore, this force may help account and construct a quantum field theory in curved space. The total force acting on a particle, in general, is thus

$$\vec{F} = m\frac{d\vec{v}}{dt} - \frac{m\vec{v}}{\tau}\,.$$

This can be compared with the force on the electron in a metal as perceived by Drude.

If we consider a force between two identical electrons separated by a distance r that is balanced by the electric force, then the angular momentum of the electron is given by

$$L=\sqrt{\alpha\left(\frac{v}{c}\right)}\,\hbar\,, \tag{22}$$

where α is the fine structure constant. If one particle rotates about the other, then its angular frequency is given by $\omega_c=\frac{mc^2}{\hbar}$. This is known as Zitterbewegung (trembling motion) oscillation exhibited by all elementary particles [9]. In quantum field theory the Zitterbewegung is caused by interference between the positive and negative energy components of the wave packet. According to eqs.(20) and (21) a particle moving with constant velocity experiences a force that may have manifested in the Zitterbewegung.

Let us now assume that the force in eq.(19) is of a Stokes' type, then the coefficient of inertial viscosity of the fluid provided by a moving charged particle (q) is given by

$$\eta=\frac{2}{3}\,\frac{mc^2}{\mu_0\hbar}\left(\frac{m}{q}\right)^2\,. \tag{23}$$

We assume here that a charged particle moving in a magnetic field is equivalent to a mass moving if a viscous fluid. We call this viscosity the magnetic viscosity. Equation (23) can be compared with the coefficient of magnetic viscosity due to motion of a charged particle in a magnetic field (B)

$$\eta=\frac{2}{3\mu_0}\left(\frac{m}{q}\right)B\,, \tag{24}$$

where we have assumed that the particle has a radius equal to its classical radius. Notice that while η in eq.(23) contains $\hbar$, it is absent in eq.(24). A critical inertial magnetic field can be obtained which makes the two coefficients equal, or making the magnetic force ($F=qvB$) and the inertial force equal is given by

$$B_{cr}=\frac{m^2c^2}{\hbar\, q}\,. \tag{25}$$

An electric field of $E_{cr}=\frac{m^2c^3}{q\hbar}$ is associated with this field. Interestingly, this is exactly equal to the critical magnetic field obtained from quantum electrodynamics by equating the cyclotron energy of the charged particle to its rest mass energy [11–14]. Now consider two particles with relative velocity v, separated by a distance r, under gravity. The particle angular momentum can be obtained by equating the inertial quantum force and the gravitational force, *viz.*,

$$L=\frac{Gm\,\hbar}{c^2r}\,. \tag{26}$$

Owing to eq.(26), a spinning particle whose radius is equal to the Schwarzschild radius ($2Gm/c^2$), its spin angular momentum is

$$S=\frac{\hbar}{2}\,. \tag{27}$$

However, eq.(26) reveals that a charged particle with a size equal to its classical radius has a spin angular momentum

$$S=\frac{G}{k}\left(\frac{m}{q}\right)^2\hbar\,,\qquad S=\frac{Gm^2}{c\,\alpha}\qquad(\text{for }\ q=e)\,, \tag{28}$$

where k is the Coulomb's constant. The spin in eq.(28) can be compared with that of a black hole of mass m which is Gm^2/c.

Similarly a charged moving particle at a distance r from another particle will have an orbital angular momentum

$$L = \frac{kq^2\hbar}{mc^2 r}\,, \qquad L = \left(\frac{r_c}{r}\right)\hbar\,. \tag{29}$$

Hence, a spinning charged particle whose radius is equal to the classical radius will have a spin angular momentum

$$S = \hbar\,. \tag{30}$$

Equation (29) shows that a particle will have spin, $S = \frac{\hbar}{2}$ if its radius is twice the classical radius.

The power radiated by an accelerating charge is given by [8]

$$P = \frac{2\,q^2}{3\,\varepsilon_0}\frac{a^2}{c^3}\,. \tag{31}$$

The self - power of a particle moving with velocity v, owing to eq.(19), is

$$P = \frac{m^2c^2v^2}{\hbar}\,. \tag{32}$$

For a charged particle moving in a circular orbit, one has $a = \frac{v^2}{r}$, and hence equating eq.(31) and eq.(32) will yield the angular frequency ($\omega = \frac{v}{r}$)

$$\omega = \left(\frac{2\pi c^3}{\mu_0 h}\right)^{1/2}\frac{m}{q}\,. \tag{33}$$

However, in a cyclotron a charged particle moves in a circular path when enters a perpendicular magnetic field, its angular frequency is given by

$$\omega = \frac{qB}{m}\,. \tag{34}$$

Equating eqs.(33) and (34) yields a limiting field

$$B_0 = \left(\frac{3\pi c^3}{\mu_0 h}\right)^{1/2}\frac{m^2}{q^2}\,. \tag{35}$$

Equation (35) can be written as

$$B_0 = \sqrt{\frac{3}{2\alpha}}\,B_{cr}\,, \tag{36}$$

for an electron or a proton. The angular frequency can be written as, $\omega = \sqrt{\frac{3}{2\alpha}}\,\frac{mc^2}{\hbar} = \sqrt{\frac{3}{2\alpha}}\,\omega_c$.

The Unruh effective temperature experienced by a uniformly accelerating (a) observer is given by [15]

$$T = \frac{\hbar\,a}{2\pi c k_B}\,, \tag{37}$$

where k_B is the Boltzmann constant. It seems that particles are created from the vacuum due to observer's acceleration. If the above acceleration is due to the quantum inertial effect, then using eq.(16a), eq.(37) yields

$$T_q = \left(\frac{mc^2}{2\pi k_B}\right)\frac{v}{c}\,. \tag{38}$$

It seems that T_q is a correction to the ordinary temperature of the radiation emitted by a moving particle. Equation (16a) can be written as

$$a=\frac{mc^2}{\hbar}\,v\,,\qquad\qquad \text{or} \qquad\qquad a=\omega_c\,v\,. \tag{39}$$

The apparently high acceleration is achieved only during the time during which the pair is created. That period can be thought of as a phase transition that occurs in thermodynamic system. And since this time is incredibly short it is utterly hard to observe this radiation in the Lab frame. If we assume $m=M_P$ (the Planck's mass), then the temperature $T_q\sim 10^{32}K$, that is the initial universe temperature (Planck's temperature), and $a=10^{51}\,\mathrm{m\,s^{-2}}$. Equation (25) suggests that the primeval magnetic field (where $m=M_P$) is $B_{cr}\sim 10^{53}\,\mathrm{T}$ (the Planck's magnetic field). Note that the electromagnetic energy density is given by

$$\rho_{em}=\frac{\pi^2}{15}\frac{(k_BT)^4}{(c\hbar)^3}\,. \tag{40}$$

Assuming that the radiation emitted by an accelerating charge follows this law, we can apply eq.(38) to obtain the quantum energy density

$$\rho_q=\rho_0\left(\frac{v}{c}\right)^4\,,\qquad \rho_0=\frac{m^4c^5}{240\,\pi^2\hbar^3}\,. \tag{41}$$

where $\rho_0=\frac{E_0}{V_c}$, $V_c=240\,\pi\lambda^3$, and $\lambda=\frac{\hbar}{mc}$. We may treat V_c as the effective volume in which the rest mass energy is extended (distributed), bearing in mind that the electron moves as a wavepacket and not as a single point. This radiation energy density, ρ_q, may result from the process of creation and annihilation accompanying the moving particle, where the particle initial mass is transformed into energy that later yields a pair. If we assume now the cosmic background radiation ($T_0=2.72\,\mathrm{K}$) is due to this radiation, then eq.(36) with $v\sim c$ yields a mass of $m=\frac{2\pi k_BT_0}{c^2}$ for the cosmic photon.

The magnetic field in the instantaneous rest frame of the moving particle is determined by $\vec{p}\,'=m\vec{v}+q\vec{A}=0$, where $\vec{A}$ is the magnetic vector potential. Hence, eq.(21) can be written as

$$\vec{F}_m=-\frac{qmc^2}{\hbar}\,\vec{A}\,. \tag{42}$$

Integrating eq.(42) with respect to dr, and using Stokes theorem yield the inertial potential energy

$$U_m=-mc^2\frac{q}{\hbar}\,\phi_B \tag{43}$$

where ϕ_B is the magnetic flux, $\phi_B=\int\vec{A}\cdot d\vec{r}$. If the flux is quantized, then $\phi_B=n\hbar/q$ so that $U_m=-n\,mc^2$, where $n=1,2,3,\cdots$.

Recall that the term $\Delta\theta=\frac{q}{\hbar}\phi_B$ defines a phase difference that the wavefunction of a charged particle will acquire when a particle traverses two paths enclosing a magnetic field [14]. This may indicate that the charged particle undergoes pair production where each subparticle gets interference with the other when they recombine in a construction (producing the initial particle) and a destruction (the energy).

Let us take the divergence of eq.(42) and use eq.(10) to obtain

$$\vec{\nabla}\cdot\vec{F}_m+\frac{\partial R_m}{\partial t}=0\,,\qquad R_m=\frac{m}{\hbar}q\varphi\,. \tag{44}$$

Here R_m can be defined as the rate of creation or annihilation of mass due to an energy imparted by the electromagnetic field in which the particle is moving. Hence, the particle converts its electromagnetic energy ($q\varphi$) by creating a pair. Therefore, eq.(44) represents an energy conservation equation.

Upon taking the curl of eq.(42) and using eqs.(9) and (19), we obtain the vorticity

$$\vec{\omega}=\frac{q}{2m}\vec{B}\,,\qquad \vec{\Omega}=\vec{\nabla}\times\vec{v}\,,\qquad \vec{\Omega}=2\vec{\omega}\,. \tag{45}$$

Equation (45) is very interesting as it relates the magnetic field to the angular velocity ($\vec{\omega}$). If $\vec{v}$ is constant, then $\vec{B}=0$. This results urges us to assume that the quantum particle moves like a rotating fluid. Notice that $\omega_L=\frac{q}{2m}\,B$ is the Larmor frequency that represents a precession of the charged particle (magnetic moment) when placed in an external magnetic field. Therefore, the moving particle precesses when placed in an external magnetic field with the value expressed in eq.(45). This also implies that the electron has a spin angular momentum.

Thus a moving massive photon inside a conducting medium will be governed by eqs.(13) and (42), where

$$\vec{\nabla}\cdot\vec{F}_m+\frac{\partial R_m}{\partial t}=-\frac{R_m}{\tau_c}\,,\qquad \tau_c=\frac{\varepsilon_0}{\sigma}\,, \tag{46}$$

where τ_c is the relaxation time of the photon inside the conductor.

Concluding Remarks. – We have introduced in this work a new formulation of quantum mechanics analogous to Maxwell formulation of electrodynamics. In this respect there exist two fields, analogous to Maxwell electric and magnetic fields, associated with the moving mass (particle). We have shown that an inertial quantum force analogous to Lorentz force emerges from this formulation that reflects the creation and annihilation process undergoes by all elementary particles. Owing to this, an inertial quantum force exists even when the particle is moving at constant velocity. The electromagnetic wave is found to acquire mass when it propagates in a conducting medium. This force is a manifestation of the inertia that all material waves undergo. Inside the conducting medium the Lorentz gauge condition is no longer valid, and consequently the gauge invariance is broken. The conductivity is found to be related to the mass the particle developed in the medium. If our acceleration is due to Unruh effect that associates a temperature with the black-body radiation that an accelerating observer would see, we conclude that $a=\omega_c v$. This suggests that such a radiation occurs only during a very short period of time, $\tau=\hbar/mc^2$. Interesting, we have found that a charged particle whose radius equals to the classical radius will have a spin of $s=\hbar$, and that with twice the classical radius has a spin $s=\hbar/2$. When the moving particle is placed in an external magnetic field, the particle precesses with Larmor frequency about the magnetic field.

REFERENCES

[1] Sciama, D. W., *On the origin of inertia,MNRAS*, **113** (1953) 34

[2] de Broglie, L., *Compt. Rend.*, **199** (1934) 813
[3] Arbab, A. I., *Derivation of Dirac, Klein-Gordon, Schrdinger, diffusion and quantum heat transport equations from a universal quantum wave equation, EPL*, **92** (2010) 40001
[4] Caldeira, A. O. and Leggett, A. J., *Path integral approach to quantum Brownian motion, Physica A*, **121** (1983) 587
[5] Caldeira, A. O. and Leggett, A. J., *Quantum tunneling in a dissipative system, Ann. Phys.*, **149** (1983) 374
[6] Arbab, A. I., *The Quaternionic Quantum Mechanics, Appli. Phys. Res.*, **3** (2011) 160
[7] Kraus, J. D., *Electromagnetics,* (McGraw-Hill,) 1984
[8] Griffiths, D., *Introduction to Electrodynamics,* (Prentice-Hall) 1999
[9] Schroedinger, E., *Sitzungber. Preuss. Akad. Wiss. Phys.-Math. Kl.*, **24** (1930) 418
[10] Dirac, P. A. M., *Proc. R. Soc. Lond. A1*, **117** (1928) 610
[11] Sauter, F., and Semionova, L., *Uber das Verhalten eines Elektrons im homogenen elektrischen Feld nach der relativistischen Theorie Diracs, Z. Phys.*, **69** (1931) 742
[12] Leahy, D., and Semionova, L., *New QED calculations for processes in strong magnetic fields, AIP Conf. Proc.*, **526** (2000) 862
[13] Kirk T. McDonald, *Limits on the Applicability of Classical Electromagnetic Fields as Inferred from the Radiation Reaction, http://arxiv.org/abs/physics/0003062*
[14] Schwinger, J., *On Gauge Invariance and Vacuum Polarization, Phys. Rev.*, **82** (1951) 664
[15] Leahy, D., and Semionova, L., *New QED calculations for processes in strong magnetic fields, Phys. Rev. D*, **14** (1976) 870