arXiv:physics/0205086v3 [physics.gen-ph] 9 May 2003

Connectivity and the Origin of Inertia

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Abstract

Newton's Second Law defines inertial mass as the ratio of the applied force on an object to the responding acceleration of the object (viz., $F = ma$). Objects that exhibit finite accelerations under finite forces are described as being "massive" and this mass has usually been considered to be an innate property of the particles composing the object. However mass itself is never directly measured. It is inertia, the reaction of the object to impressed forces, that is measured. We show that the effects of inertia are equally well explained as a consequence of the vacuum fields acting on massless particles travelling in geodesic motion. In this approach, the vacuum fields in the particle's history define the curvature of the particle's spacetime. The metric describing this curvature implies a transformation to Minkowski spacetime, which we call the Connective transformation. Application of the Connective transformation produces the usual effects of inertia when observed in Minkowski spacetime, including hyperbolic motion in a static electric field (above the vacuum) and uniform motion following an impulse. In the case of the electromagnetic vacuum fields, the motion of the massless charge is a helical motion that can be equated to the particle spin of quantum theory. This spin has the properties expected from quantum theory, being undetermined until "measured" by applying a field, and then being found in either a spin up or spin down state. Furthermore, the zitterbewegung of the charge is at the speed of light, again in agreement with quantum theory. Connectivity has the potential for pair creation as the Connective transformation can transform positive time intervals in the particle spacetime to negative time intervals in Minkowski spacetime.

1 Introduction

Renewed interest in explaining the origin of inertia has been triggered by the foundational work of Rueda, Haisch, and Puthoff [1,2]. Rather than accepting that inertial mass is simply an innate property of matter as defined in Newton's Second Law, these authors show that the effects of inertia can be explained as a consequence of accelerated motion in the electromagnetic vacuum field, the so-called Zero-Point Field (ZPF). We develop a formal approach to the explanation of inertia as a reaction to the vacuum fields that simultaneously addresses aspects of quantum theory that heretofore have been problematic from a Classical particle point of view. Specifically we address the quantum-theoretic prediction that the zitterbewegung of particles occurs at the speed of light, that particles exhibit spin, and that pair creation can occur.

In his study of the coordinate operator in the Dirac equation, Schrödinger discovered microscopic oscillatory motion at the speed of light, which he called *zitterbewegung* [3]. While Dirac argued that such motion does not violate relativity or quantum theory [4], from a Classical particle point of view, these speed of light motions would seem to imply masslessness of the particle.

Dirac theory also describes particle spin, and Schrödinger considered spin to be an orbital angular momentum that is a consequence of the vacuum fields [3]. This view of spin was explored further by Huang [5] and Barut and Zanghi [6].

We present a description of the motion of massless particles that manifests inertia as a consequence of the vacuum fields. This approach simultaneously yields spin as an orbital angular momentum driven by the electromagnetic vacuum (ZPF). We derive an equation of motion for massless charges as a limit of the Lorentz force equation. This equation can be viewed as a geodesic equation in a spacetime associated with the particle, and the implied transformation to Minkowski spacetime (the Connective transformation) yields the effects of inertia as viewed in Minkowski spacetime. A massless charge is seen in Minkowski spacetime to be driven by the ZPF in helical motions that we interpret to be spin. The theory has the potential to exhibit the many-particle behavior of pair creation and annihilation as the Connective transformation sometimes takes positive time intervals in the particle spacetime to negative time intervals in Minkowski spacetime.

We will formulate Connectivity in the context of electromagnetism, where the particle is assumed to be a massless electric charge and the vacuum fields are the ZPF. The approach should be generalizable to the other vacuum fields (e.g., the color fields of the strong nuclear force).

In the following we use the metric signature $(-,+,+,+)$.

2 Massless charge equation of motion

A suitable equation for describing the motion of a massless charge can be derived as the massless limit of the Lorentz force equation,

$$m\alpha^{\mu} = \frac{q}{c} F^{\mu\nu} u_{\nu} \quad , \tag{1}$$

where q is the charge of the particle, c is the speed of light, and m is the particle mass (which we will soon take to be zero). $F^{\mu\nu}$ is the electromagnetic field tensor of the impressed fields, including the ZPF. The acceleration α and the velocity u are four-vectors. In terms of the usual three-space acceleration $\mathbf{a}$ and velocity $\mathbf{v}$ we have

$$m\left[\gamma^2 a^j + \gamma^4 \mathbf{v}\cdot\mathbf{a}\frac{v^j}{c}\right] = \frac{q}{c}\gamma\left[\sum_{k=1}^{3} F^{jk} v_k - F^{j0} c\right] \tag{2}$$

$$m\gamma^4 \frac{\mathbf{v}\cdot\mathbf{a}}{c} = \frac{q}{c}\gamma\left[\sum_{k=1}^{3} F^{0k} v_k - F^{00} c\right] \quad , \tag{3}$$

where $\gamma = 1/\sqrt{1 - v^2/c^2}$. These equations can be combined to yield

$$m\gamma a^j = \frac{q}{c}\left[\sum_{k=1}^{3}\left(F^{jk} - F^{0k}\frac{v^j}{c}\right)v_k - \left(F^{j0} - F^{00}\frac{v^j}{c}\right)c\right] \quad . \tag{4}$$

We take the massless limit to be that in which the mass goes to zero as the speed of the particle becomes the speed of light (hence $\gamma \to \infty$). The product $m\gamma$ is assumed to remain finite in this limit, that is,

$$\lim_{\substack{m \to 0 \\ v \to c}} m\gamma = m_* \quad . \tag{5}$$

The mass parameter m_* has the dimensions of mass, but it is *not* mass. Particles described by (5) move at the speed of light and therefore must satisfy

$$\mathbf{v} = c\mathbf{n} \quad , \tag{6}$$

where $\mathbf{n}$ is a unit vector in the direction of the particle motion. Accelerations therefore can only be due to changes in the direction of the particle, not in its speed,

$$\mathbf{a} = c\frac{d\mathbf{n}}{dt} \quad . \tag{7}$$

We have, then,

$$m_* c\frac{dn^j}{dt} = \frac{q}{c}\left[\sum_{k=1}^{3}\left(F^{jk} - F^{0k}\frac{v^j}{c}\right)v_k - \left(F^{j0} - F^{00}\frac{v^j}{c}\right)c\right] \quad . \tag{8}$$

In terms of the electric and magnetic fields, this is

$$\frac{d\mathbf{n}}{dt} = \frac{q}{m_* c}\left[\mathbf{n} \times \mathbf{B} - \mathbf{n}\cdot\mathbf{E}\mathbf{n} + \mathbf{E}\right] \quad . \tag{9}$$

Note in (9) that only transverse forces accelerate the particle, consistent with conditions (6) and (7). This is also consistent with the expectations of relativity. Since it is moving at the speed of light, the particle should see a universe Lorentz contracted to two transverse dimensions, so it can only be accelerated by forces from the side.

Equation (8) describes the motion of a massless charge in response to impressed electromagnetic fields. The charge moves at a constant speed (the speed of light) with a changing direction given by (8). When the impressed fields include the ZPF, this motion may be regarded as Schrödinger's zitterbewegung. When a field above the vacuum is applied, the charge will be observed to drift in a preferred direction in its Zitterbewegung wander.

These effects are illustrated in Figures 1 and 2. Figure 1 shows the trajectory of a massless charge computed using equation (8). The electromagnetic fields influencing the motion of the charge are a random realization of the ZPF (see Appendix A for a description of how the random realization is generated) with a superimposed uniform electric field in the vertical direction (the driving field), which is switched off after half of the total time duration of the simulation. Note that the charge drifts upward in response to the driving field. Figure 2 shows the region of Figure 1 near the origin. Here we see that the ZPF drives the charge in a pseudo-helical motion as in Schrödinger's orbital angular momentum explanation for spin.

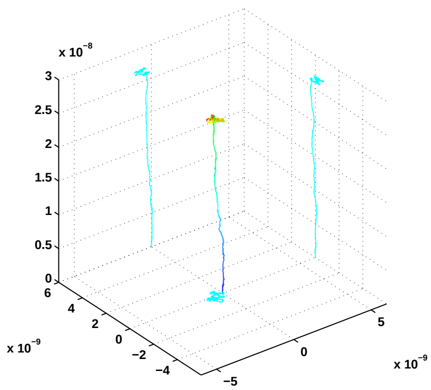

Figure 1. Massless charge trajectory in uniform electric field (plus ZPF), which is then switched off after half the total time duration.

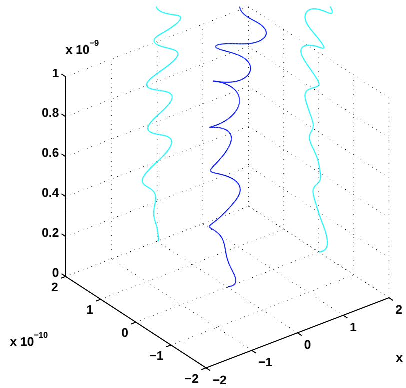

Figure 2. Zoomed view near the origin of Figure 1 showing spin-like orbital motion driven by the ZPF.

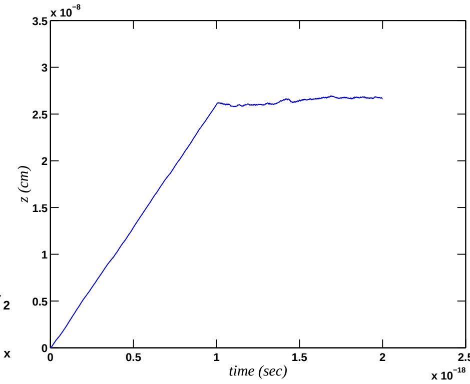

Figure 3. Drift in z-direction of Figure 1 exhibiting non-inertial behavior as charge stops drifting when driving field is switched off.

Now equation (8) by itself does not exhibit inertia. As is illustrated in Figure 1 and more clearly in Figure 3, when the driving field is switched off, the charge stops drifting and resumes a zero-mean random walk in the frame of the calculation. This non-inertial behavior is at distinct odds with relativity.

3 The Connective transformation

The non-inertial behavior of the massless charge equation of motion (8) is in distinct violation of relativity. The ZPF vacuum fields have the energy density frequency spectrum [7]

$$\rho(\omega)d\omega = \frac{\hbar\omega^3}{2\pi^2 c^3} d\omega \quad . \tag{10}$$

The cubic frequency dependence of the ZPF spectrum endows it with Lorentz invariance; all inertial frames see an isotropic ZPF. A Lorentz transformation will cause a Doppler shift of each frequency component, but an equal amount of energy is shifted into and out of each frequency bin. When there are no fields above the vacuum in an inertial frame, an observer in that frame should expect to see a zero-mean random walk due to the isotropic ZPF. Thus, in the example cited above, an observer in a frame co-moving with the average motion of the charge just before the driving field is switched off should expect to see continued zero-mean zitterbewegung in his frame, whereas (8) produces zero-mean motion in whatever frame the calculation is performed in. To have a consistent theory, Lorentz covariance must be restored.

We assume that (8) holds in the spacetime of the particle. We further assume that in this spacetime, (8) is the equation of a null geodesic. The curvature is defined by the electromagnetic fields in the particle's history. Since the charge is assumed to be massless and moving at the speed of light, we cannot use proper time as the affine parameter of the geodesic (proper time intervals vanish for null geodesics). However, normal time serves well as an affine parameter. Thus we replace (8) with

$$\frac{dp^\mu}{dt} + \frac{1}{m_*}\Gamma^\mu_{\nu\rho}p^\nu p^\rho = 0 \quad , \tag{11}$$

where

$$p^\mu = m_* c n^\mu \quad , \tag{12}$$

and

$$n^\mu = (n^0, \mathbf{n}) \quad . \tag{13}$$

$\Gamma^\mu_{\nu\rho}$ are the Christoffel symbols of the second kind. We equate the connection terms (the terms containing the Christoffel symbols) with the Lorentz force terms of equation (1). That is, we set

$$\Gamma^\mu_{\nu\rho}p^\nu p^\rho = -\frac{q}{c}F^\mu_\nu p^\nu \quad . \tag{14}$$

These equations can be solved for the metric of the particle's spacetime, though not uniquely. The equations (14) actually define a class of metrics. Further constraints are required to select a particular solution from this class. In particular, the geodesic in the particle spacetime should be a null curve as expected for a massless object. Using (14), the geodesic equation (11) can be written as,

$$\frac{dn^j}{dt} = \frac{q}{m_* c}\left[F^j_\nu n^\nu - F^0_\nu n^\nu \frac{n^j}{n^0}\right] + \frac{n^j}{n^0}\frac{dn^0}{dt} \tag{15}$$

$$\frac{dm_*}{dt} = \frac{q}{c}F^0_\nu \frac{n^\nu}{n^0} - \frac{m_*}{n^0}\frac{dn^0}{dt} \quad . \tag{16}$$

In general n^0 does not retain a value of unity, but should change in a way that preserves the null curve property,

$$g_{\mu\nu}p^\mu p^\nu = 0 \quad . \tag{17}$$

Note that (16), which is the zeroth equation of (11), is an equation for the parameter m_*. Thus m_* is not a constant, but rather varies in response to applied forces. The effect is to introduce time dilation (or Doppler shifting) in the energy-momentum four vector analogous to the gravitational redshift of General Relativity.

Also note that inertia is not assumed here by our requirement that the particle travel on a null geodesic. We do not restrict the motion of the particle, rather we simply use its motion to define the spacetime and metric that the particle sees. It is only after the transformation to Minkowski spacetime that inertial behavior appears.

To find solutions of (14), we first consider an infinitesimal region about the charge, and require

$$g_{\mu\nu} = \eta_{\mu\nu} + g_{\mu\nu,\rho}dx^\rho \quad , \tag{18}$$

where $\eta_{\mu\nu}$ is the flat spacetime metric,

$$\eta_{\mu\nu} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad . \tag{19}$$

The Christoffel symbols are then calculated in terms of the derivatives $g_{\mu\nu,\rho}$ and substituted into (14), resulting in an underdetermined set of equations for the metric derivatives. One solution for the metric tensor (valid for an infinitesimal patch about the position of the charge) is,

$$\begin{aligned}
g_{00} &= -1 + \frac{q}{m_* c^2 n_0}(E_x dx + E_y dy + E_z dz) = -1 + F_L dt \\
g_{01} &= -\frac{q}{2m_* c^2 n_0}(E_x cdt + B_z dy - B_y dz) = -\frac{1}{2}F_x dt \\
g_{02} &= -\frac{q}{2m_* c^2 n_0}(E_y cdt + B_x dz - B_z dx) = -\frac{1}{2}F_y dt \\
g_{03} &= -\frac{q}{2m_* c^2 n_0}(E_z cdt + B_y dx - B_x dy) = -\frac{1}{2}F_z dt \\
g_{11} &= g_{22} = g_{33} = 1 \\
g_{12} &= g_{13} = g_{23} = 0 \quad ,
\end{aligned} \tag{20}$$

with the remaining terms given by the symmetry of $g_{\mu\nu}$. Here we use the normalized force $F = \textit{force}/m_* c n_0$, and F_L is the longitudinal component with respect to the direction of particle motion. The second equalities in (20) hold only on the particle geodesic.

The metric $g_{\mu\nu}$ implies a transformation to Minkowski spacetime. The transformation C_μ^ν from the particle's spacetime to Minkowski spacetime is related to the metric $g_{\mu\nu}$ by $g_{\mu\nu} = C_\mu^\rho C_\nu^\sigma \eta_{\rho\sigma}$, or

$$g = C \cdot \eta \cdot \tilde{C} \quad , \tag{21}$$

where $\tilde{C}$ is the transpose of C. For the metric (20) we find (for transformations along the particle geodesic),

$$C_{\textit{infinitesimal}} = \begin{bmatrix} 1 - F_L dt & \frac{1}{4}F_x dt & \frac{1}{4}F_y dt & \frac{1}{4}F_z dt \\ -\frac{1}{4}F_x dt & 1 & 0 & 0 \\ -\frac{1}{4}F_y dt & 0 & 1 & 0 \\ -\frac{1}{4}F_z dt & 0 & 0 & 1 \end{bmatrix} \quad . \tag{22}$$

A general finite transformation can be obtained by repeated application of the infinitesimal one. In the infinite limit this yields,

$$\begin{aligned}
C_0^0(t) &= \frac{1}{F_T} \exp\left(-\frac{1}{4}\int_0^t F_L(t')dt'\right) \left[F_T \cos\left(\frac{1}{4}\int_0^t F_T(t')dt'\right) - F_L \sin\left(\frac{1}{4}\int_0^t F_T(t')dt'\right)\right] \\
C_0^i(t) &= -\frac{1}{F_T} \exp\left(-\frac{1}{4}\int_0^t F_L(t')dt'\right) F_i \sin\left(\frac{1}{4}\int_0^t F_T(t')dt'\right) \\
C_i^i(t) &= \frac{1}{F^2 F_T} \left\{F_T(F^2 - F_i^2)\right.
\end{aligned}$$

$$
\begin{aligned}
& + \quad \exp\left(-\frac{1}{4}\int_0^t F_L(t')dt'\right) F_i^2 \left[F_T \cos\left(\frac{1}{4}\int_0^t F_T(t')dt'\right) + F_L \sin\left(\frac{1}{4}\int_0^t F_T(t')dt'\right)\right]\Bigg\} \\
C_i^j(t) \quad &= \quad \frac{F_i F_j}{F^2 F_T}\{-F_T \\
& + \quad \exp\left(-\frac{1}{4}\int_0^t F_L(t')dt'\right) \left[F_T \cos\left(\frac{1}{4}\int_0^t F_T(t')dt'\right) + F_L \sin\left(\frac{1}{4}\int_0^t F_T(t')dt'\right)\right]\Bigg\} \qquad (23)
\end{aligned}
$$

where $i, j = 1, 2, 3$, $C_i^0 = -C_0^i$, $C_j^i = C_i^j$, and F_T is the magnitude of the transverse component of the normalized force (transverse with respect to the direction of particle motion). The metric $g_{\mu\nu}$ of the particle's spacetime (on the geodesic) is obtained through (21),

$$
\begin{aligned}
g_{00}(t) \quad &= \quad -\frac{1}{F_T}\exp\left(-\frac{1}{2}\int_0^t F_L(t')dt'\right)\left[F_T \cos\left(\frac{1}{2}\int_0^t F_T(t')dt'\right) - F_L \sin\left(\frac{1}{2}\int_0^t F_T(t')dt'\right)\right] \\
g_{0i}(t) \quad &= \quad -\frac{1}{F_T}\exp\left(-\frac{1}{2}\int_0^t F_L(t')dt'\right) F_i \sin\left(\frac{1}{2}\int_0^t F_T(t')dt'\right) \\
g_{ii}(t) \quad &= \quad \frac{1}{F^2 F_T}\left\{F_T(F^2 - F_i^2)\right. \\
& + \quad \exp\left(-\frac{1}{2}\int_0^t F_L(t')dt'\right) F_i^2 \left[F_T \cos\left(\frac{1}{2}\int_0^t F_T(t')dt'\right) + F_L \sin\left(\frac{1}{2}\int_0^t F_T(t')dt'\right)\right]\Bigg\} \\
g_{ij}(t) \quad &= \quad \frac{F_i F_j}{F^2 F_T}\{-F_T \\
& + \quad \exp\left(-\frac{1}{2}\int_0^t F_L(t')dt'\right) \left[F_T \cos\left(\frac{1}{2}\int_0^t F_T(t')dt'\right) + F_L \sin\left(\frac{1}{2}\int_0^t F_T(t')dt'\right)\right]\Bigg\} \qquad (24)
\end{aligned}
$$

with the remaining terms given by symmetry. As we said, the solution (23) and (24) is not unique, but it is in some sense the simplest one as it yields the equality of the connection terms and the Lorentz forces in a term-by-term manner. We will show in the next section that this metric has several desirable properties, particularly the generation of both inertia and spin. It also implies a many-particle theory with pair creation and annihilation.

One interesting aspect of (24) is that it depends on the particle history through the time integrations. This means that the spacetime of a particle is unique to it; two particles at the same place at the same time will be in different spacetimes if the history integrals of (24) differ (as they generally will).

A second interesting point is that the integration over longitudinal forces appears within exponentials whereas the integration over transverse forces appears within oscillatory trigonometric functions. The latter may account for pair creation and annihilation. The former produces time dilation that, as we shall soon see, yields the effect we know as inertia.

4 Inertia

We have shown in Figure 3 that the massless charge equation of motion (8) does not contain the effect of inertia. The formulation of the preceding section restores Lorentz covariance to the theory through the geodesic equation (11), and thus we expect that the well-known effects of inertia should appear. Two of the simplest manifestations of inertia are hyperbolic motion of a charge in the presence of a uniform electric field and uniform motion following an impulse. We now demonstrate these effects.

Figure 4 displays the component of the motion of a charge in the direction of a uniform electric field applied above the ZPF, obtained by solving (11) using the metric (24) and applying the Connective transformation to view the result in Minkowski spacetime. The solid curve is the result of the Connectivity simulation. The circles lie on the hyperbola defined by a massive charge undergoing uniform acceleration in special relativity. The agreement is striking. Viewed in Minkowski spacetime, the massless charge is seen to accelerate hyperbolically as though it has inertia. Figure 5 is a similar depiction for the case in which an impulse has been applied to the charge, and the charge is observed in Minkowski spacetime to continue in uniform motion following the impulse. Here the circles lie on a straight line, indicating that the particle travels at a constant speed following the impulse. Note that it is the average motion of the charge that moves uniformly. Deviations about the average motion are apparent and are, in fact, zitterbewegung driven by the ZPF. The average motion defines a timelike curve, as expected of a massive particle. (That the metric (24) produces instantaneous motion on a *null* curve needs to be verified since our equations *assume* a massless particle. From our numerical simulations, this appears to be at least approximately true for the single-particle regime.)

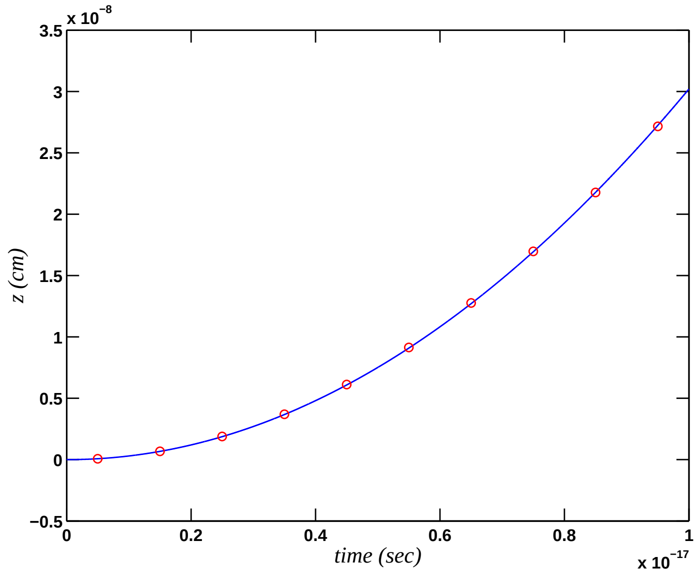

Figure 4. Massless charge motion in a uniform electric field (plus ZPF), obtained using Connectivity (solid curve), compared to the hyperbolic motion of a massive charge in special relativity (circles).

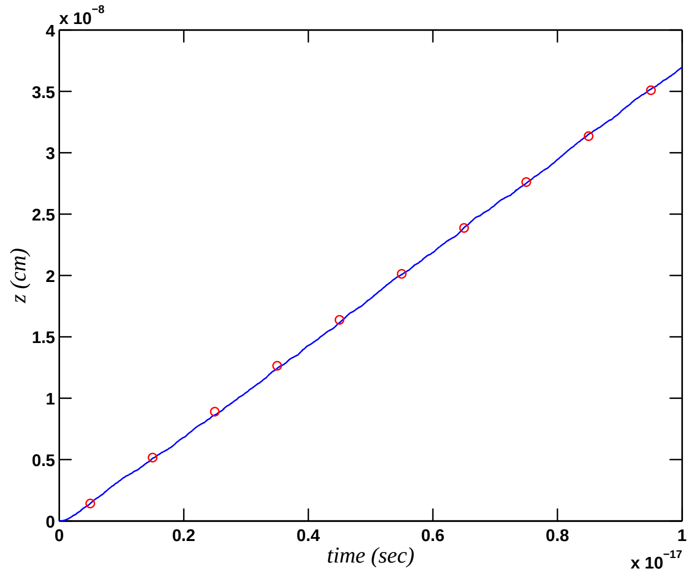

Figure 5. Massless charge motion in the ZPF following an impulse, obtained using Connectivity (solid curve), compared to constant speed motion (circles).

In the simulations of Figures 4 and 5, we were careful to require that the charge was never energized enough for particle pair creation to occur. This was done by initiating the simulation with a sufficiently large value of m_*. As is apparent in (23), if the particle is energized sufficiently by the ZPF plus driving field, the Connective transformation can cause positive time intervals in the particle's spacetime to be transformed to negative time intervals in Minkowski spacetime. This negative-time transport of the particle can be viewed as the positive-time transport of its antiparticle. The multi-particle regime was avoided here so that the inertia-generating aspect of Connectivity could be demonstrated without complication.

5 Discussion

The premise of the Connectivity approach is that charges are massless and move on null geodesics in a spacetime whose curvature is defined by equating the forces on the charge with the connection terms in the geodesic equation. These forces include those of the underlying vacuum fields. The metric of this spacetime defines a transformation to Minkowski spacetime, wherein the charge is observed to exhibit inertial behavior.

The simple fact that the Connectivity premise yields the effects of inertia from massless charges is strong motivation to seriously consider other implications of the theory, especially with regard to its potential to shed light on the basis of the quantum theory.

When the forces acting on the charge are the Lorentz forces due to the electromagnetic vacuum fields (the ZPF), these drive the charge in zitterbewegung motion at the speed of light, in agreement with the speed-of-light eigenvalues of the Dirac theory. When the charge moves with a large average velocity in some direction, the zitterbewegung motion extends to a quasi-helical motion that may be the basis of particle spin. This spin is undetermined until "measured" by applying a field that aligns the zitterbewegung into helical motion, which will either be oriented with positive or negative helicity (spin up or spin down). Further work will be required to determine whether it is possible for this "spin" to manifest the spin correlation properties of the quantum theory.

It is implied by the Connective transformation (23) that the charge can be observed in Minkowski spacetime to be travelling backward in time, or rather that its antiparticle is travelling forward in time. Thus we have the basis for a many particle theory that manifests the degrees of freedom of the Dirac theory (positive and negative energy states, each with spin up/down configurations).

There are a number of directions for further research that are implied by the Connectivity premise. These include studying the correspondence of Connectivity with quantum theory, the theory of electromagnetic radiation, and with gravitation theory.

For example, we suspect that it might be possible to derive the Schrödinger and Dirac equations as stochastic equations obtained from the ensemble average of the Connective transformation applied to the geodesic equation (11) in the single- and multi-particle regimes respectively.

Since the interpretation of Connectivity is that electromagnetic fields define a curvature of spacetime, it may be possible to show that radiation is related to the distortion of spacetime required to connect the views of different frames. This might then have implications for the problem of the stability of atoms.

Exploring the relationship between the particle spacetime of Connectivity (which generates inertia) to the spacetime of General Relativity may shed light on Einstein's postulate of the Equivalence Principle. Among the things assumed in General Relativity are *i.)* the Equivalence Principle, *ii.)* a spacetime with a metric which is well defined even in the absence of matter and energy, *iii.)* matter and energy, which are already assumed to have well defined rest masses and only distort the metric. Given these (with appropriate topology and boundary conditions), the inertial frames are well defined. However, the standard version of relativity does not explain why the Equivalence Principle holds in the first place, how the metric can be well defined in an empty spacetime (how do we measure motion without reference points?), and what are the rest masses of the individual particles (or the weight of an object held up against gravity). In each case, the solution of these problems may lie in considering the effect of the vacuum as well as that of the matter and energy above the vacuum. Even in an "empty" spacetime, vacuum fields can provide spacetime events to which motion can be referred and acceleration with respect to the vacuum can be detected. Secondly, according to Connectivity, interaction with the vacuum endows individual particles with inertia and a corresponding effective mass, possibly providing a means of determining the individual rest masses. Finally, if we can derive inertial effects from interaction with the vacuum, it seems plausible that we should be able to similarly derive gravitational effects, thereby yielding the equivalence principle and the Minkowski metric. Explicit reference to (and interaction with) a real vacuum may provide reasonable explanations for the fundamental assumptions of gravitation theory as represented in General Relativity.

6 Appendix A: ZPF Realizations

A convenient way to explore the properties of the massless charge equation of motion (8) and the Connective transformation (23) is through numerical simulation. This requires the generation of ZPF realizations. We use the prescription of Ibison and Haisch [8],

$$\mathbf{E}(\mathbf{r},t) = Re\int d^3k \sum_{\lambda=1}^{2} \hat{\epsilon}_{k,\lambda} \frac{\sqrt{\hbar\omega}}{2\pi}(u_{k,\lambda} + iv_{k,\lambda}) \exp\left[i(\mathbf{k}\cdot\mathbf{r} - \omega t)\right] \tag{25}$$

$$\mathbf{B}(\mathbf{r},t) = Re\int d^3k \sum_{\lambda=1}^{2} \hat{k} \times \hat{\epsilon}_{k,\lambda} \frac{\sqrt{\hbar\omega}}{2\pi}(u_{k,\lambda} + iv_{k,\lambda}) \exp\left[i(\mathbf{k}\cdot\mathbf{r} - \omega t)\right] \quad . \tag{26}$$

Here $u_{k,\lambda}$ and $v_{k,\lambda}$ are Gaussian random numbers with zero mean and unit variance, $\mathbf{k}$ is the wave vector of a plane wave component of the ZPF, and λ is the polarization index. Gaussian electromagnetic units have been used.

In generating random realizations of the ZPF, two practical considerations are forced upon us. Although the ZPF may theoretically extend to infinite frequencies, we must assume a cutoff frequency to build a numerical realization. There are physical reasons for applying a cutoff frequency, not in the ZPF *per se*, but in the charge's response to the ZPF. This is related to our spin interpretation of the orbital angular momentum of the helical ZPF-driven motion of the charge. The value of this angular momentum is dependent on the presumed cutoff frequency, and for the electron this implies a cutoff in the vicinity of the electron Compton frequency. The simplest way to visualize this cutoff is to assume that the charge has a finite spatial extent and is not, in fact, a point singularity. It is then only influenced by the portion of the ZPF spectrum with wavelengths longer than the charge distribution size. The second consideration affecting our random realizations is related to the size of the realization. Ideally one would generate a complete space-time realization using (25) and (26), but since we want to consider frequencies up to a cutoff frequency near the electron Compton frequency and also consider comparatively long time durations, computational memory storage becomes an issue. In order to be able to run the simulations on a very modest computer, we make the following compromise. We generate a time-history realization at the point $\mathbf{r} = 0$, but apply this realization to the charge no matter where it is. Since the realization has the correct spectral properties in frequency, this approximation can be expected to correctly give most of the behaviors we seek to demonstrate, but we must realize that certain spatial correlation aspects may not be correctly represented. Thus, for example, we see an orbital angular momentum emerging in our realizations of the charge motion that we interpret to be spin, but if one wants to simulate spin correlation effects for comparison to the predictions of the quantum theory, the spatial aspect of (25) and (26) should be included.

Acknowledgement

The work of Dr. Nickisch was supported in part by the California Institute for Physics and Astrophysics.

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