Inertial Mass and Vacuum Fluctuations in Quantum Field Theory

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Abstract

After recalling the different ways to define inertial mass of elementary particles in modern physics, we study the relationship between the mass of charged particles and zero-point electromagnetic fields. To this end we first introduce a simple model comprising a scalar field immersed in stochastic or thermal electromagnetic fields. Then we sketch the main steps of Feynman mass renormalization procedure. Our approach is essentially pedagogical and in line with the standard formalism of quantum field theory, but we also try to keep an open mind concerning the physical interpretation. We check, for instance, if it is possible to start from a zero bare mass in the renormalization process and express the finite physical mass in terms of a cut-off. Finally we briefly recall the Casimir-induced mass modification of conducting or dielectric bodies.

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1 Introduction

The concept of elementary particle is central in modern physics. Each elementary particle is characterized by a few parameters which define essentially its symmetry properties. Mass and spin define the behavior of the particle wavefuction with respect to spacetime (Poincarè) transformations; electric charge, barion or lepton number etc. define its behavior with respect to gauge transformations. These same parameters also determine the gravitational and gauge interactions of the particle.

Unlike spin and charge, mass is a continuous parameter which spans several magnitude orders in a table of the known elementary particles. We shall regard as elementary those particles which do not exhibit any internal structure up to the highest available scattering energies – even though this distinction has shifted in the past and will probably do again. Presently all the components of the standard model are considered as elementary in the physics community.

The inertial mass of a particle is in principle determined through its dynamic response to an external force. Let us refer to this for brevity as the "mass spectrometer method". According to

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quantum mechanics, however, the mass is exactly determined only for stable particles; for particles with a finite lifetime τ, there is an intrinsic indetermination $\Delta m \simeq \hbar\tau^{-1}/c^2$. In practice, the mass of short-living particles (τ lepton, $W^{\pm}$ and Z^0 bosons, heavy quarks) is not determined through mass spectrometers, but in an indirect way, namely by measuring the resonance energy in their production cross-section. This is a typical concept of quantum field theory (QFT) and we shall return to it later, relating the particle mass to the pole in its Feynman propagator.

There are also elementary particles (the light quarks) which can only be observed in states with bound energy comparable to the particles' masses. In this case, the mass of the "free" particles remains quite undefined or depends strongly on the model employed – for instance, the parton model.

Among massive elementary particles (thus excluding composite hadrons) only the Z^0 boson is uncharged. The case of the neutrino is still unclear. The most recent data on ν_μ/ν_e oscillations indicate a disappearance rate of ν_μ compatible with a mass $\Delta m^2 \sim 10^{-3}$ to 0.1 eV^2 [1].

Before coming to the specific scope of this paper, let us recall in brief some other topics connected to mass.

1.1 Effective mass

The mass spectrometer method allows to check experimentally that the mass of a particle depends on its "environment" and its interactions. For instance, an electron reacts differently to an external magnetic field when it is in the vacuum or in a Bloch state ψ_k inside a crystal. The effective mass of an electron in a lattice can be generally defined in terms of the second derivative of $E(k)$ with respect to k:

$$\frac{1}{m^*} = \frac{1}{\hbar^2}\frac{d^2E(k)}{dk^2} \tag{1}$$

We see here an equivalent definition of mass in terms of a "relation dispersion", the relation between energy and momentum of the particle. In semiconductors the behavior of $E(k)$ near the band gaps is such that the effective mass is reduced to 0.01-0.1 of the free electron mass; in certain ceramic compounds, on the contrary, the effective mass can be 5-6 times larger than the free electron mass. This means that electrons inside the material respond to applied external fields as if their mass were m^* (without any dissipation). The physical reason for this kind of mass modification is well known and resides in the interaction between the electrons and the crystal lattice. Widely used, in semiconductors, for the determination of the effective mass is cyclotron resonance with centimeter or millimeter wave radiation; the resonance frequency is given by $\omega = eB/m^*$.

In certain cases there can be interaction of the particle with the environment, still the mass does not change due to some symmetry. An example is the photon mass in the Dirac quantum vacuum, rich of e^+/e^- virtual pairs. The combination of Lorentz and abelian gauge symmetry keeps the photon massless to any order in perturbation theory. In the presence of a scalar field, however, the gauge symmetry can break down spontaneously, photons acquire a mass and the electromagnetic field becomes short-ranged. This mechanism causes the Meissner effect in condensed matter and its relativistic analogue in the theory of electroweak interactions (compare Section 3.2).

1.2 Microscopic models for elementary particles

There have been several attempts is the history of physics to express the masses of elementary particles in terms of fundamental constants. These attempts were all based upon some "microscopic" model for the particles. The classical electromagnetic model for the electron assumed that its mass was entirely due to electrostatic energy, thus giving a radius $r_e \sim e^2/m_e c^2 \sim 10^{-13}$ cm, the "classical radius" (in the following we shall employ natural units in which $c = \hbar = 1$; in these units $e^2 = \alpha \simeq 1/137$; it is clear from this formula that $m_e \sim 10^{10}$ cm^{-1} in these units).

The subsequent development of this model at the times of early quantum mechanics and QFT is well described in the book by Milonni [2], including the idea that the electrostatic repulsion between the various "parts" of the electron could be compensated by a Casimir attraction. As a matter of fact, however, the electron looks pointlike down to the shortest distances which can be explored in particle accelerators.

Further microscopic models for elementary particles and mass originated from gravitation and string theory (see [3] and ref.s). For instance, the Kerr solution of Einstein equations in General Relativity represents the gravitational field of a rotating black hole. In a certain range of the parameters, it displays some features indicating a relation to the structure of the spinning elementary particles. For a special choice of these parameters, one can obtain a model for the electron, with charge, mass, spin, magnetic moment, and a giromagnetic ratio which is automatically the same as predicted by the Dirac equation [4]. More recent work has concentrated on Kaluza-Klein theory [5].

In 1992 the Kerr solution was generalized by Sen to low energy string theory [6]. It was shown that black holes can be considered as fundamental string states, and the point of view has appeared that some of them can be treated as elementary particles [7].

The idea that all the different elementary particles can be excitation modes of a single string-like object is clearly quite appealing, and could solve the problem of the masses; however, a generally accepted model is still lacking.

1.3 Outline of this work

In this paper we offer some reflections on the concept of inertial mass in contemporary physics. We shall illustrate in a simple way (the only pre-requisite being that the reader is familiar with quantum mechanics and elementary field quantization) how the mass of a particle can be influenced by the fluctuations of the fields with which the particle interacts. Such fluctuating fields (for instance, electromagnetic fields) can be regarded (1) as classical (commuting) stochastic or thermal fields, or (2) as quantum fields, satisfying the canonical commutation relations. These two cases are generally different, though connected by several similarities and partial equivalence relations (see the books by Milonni [2] and La Pena and Cetto [8]).

In Section 2, using a scalar field model, we shall compute the effective mass of charged particles moving in an external thermal or stochastic electromagnetic field. For particles described by a scalar field ϕ, the squared mass is the coefficient of the $|\phi|^2$-term in the lagrangian density. In Section 2.1 we explain in an elementary way this property of field theory, which can also be more generally expressed by saying that the particle mass is "the pole in its Feynman propagator" (see Section 4.1). After getting used to this translation of the concept of mass, one easily checks that the correction to the squared mass is of the form $\Delta m^2 = e^2 \langle |\mathbf{A}(x)|^2 \rangle$. The average value is meant in a thermal or stochastic sense; it is straightforward to relate this average to the thermal radiation spectrum or, in the Stochastic Electrodynamics case, to the Lorentz invariant vacuum spectrum. In the thermal case the mass correction is finite, depends on the temperature and is generally very small. In the stochastic case the correction is formally infinite, unless some frequency cut-off is introduced.

In Section 3 we observe that if the electromagnetic field is regarded as a quantum field, then the above expression for the mass correction does not make sense, because it involves the square of the field – a quantity which diverges in QFT as necessary consequence of the commutation rules of the theory and unitarity. This remark gives us the occasion to recall very briefly the main features of QFT and its fundamental role in the 20. century physics, its successes and longstanding problems, and the attempts to modify it by introducing a discretization procedure which cures the infinities while being at the same time "natural" and not arbitrary.

In Section 4 we recall the main steps of the so-called Feynman mass renormalization procedure. We consider a scalar field with quartic self-interaction and the one-loop contribution to its self-energy $\Sigma(p^2)$. Expanding $\Sigma(p^2)$ from an arbitrary point m^2, we find the equation $m_0^2 + \Sigma(m^2) = m^2$, which relates the "bare" mass m_0 to the physical mass m. We also give the analogous relations valid for QED and scalar QED.

In Section 5 we discuss whether it is possible to set $m_0 = 0$ or m_0 finite (instead of infinite) in the relation above, and solve for m as a function of a suitable cut-off in $\Sigma(p^2)$.

Finally in Section 6 we recall the concept of Casimir energy of a conducting or dielectric body and its contribution to the body's inertia.

2 Effective mass of charged particles in a thermal or stochastic electromagnetic field

Let us consider charged particles with "bare" mass m_0 (i.e., the hypothetical mass of the non-interacting particles), described by a scalar field ϕ, and immersed in a thermal or stochastic electromagnetic background $A_\mu(x)$. The dynamics of each single particle can be derived from the Lagrangian of the field ϕ. In Section 2.1 we show that the coefficient of $|\phi|^2$ in the Lagrangian corresponds to the squared mass. (This is a special case of the property explained in Section 4.1.) Then, in Section 2.2 we compute this coefficient by averaging over the electromagnetic field.

2.1 Squared mass as the coefficient of $|\phi|^2$

The Lagrangian density for charged scalar particles coupled to the electromagnetic field $A_\mu(x)$ is of the form

$$L = \frac{1}{2}\phi^*(P^\mu - eA^\mu)\phi(P_\mu - eA_\mu) - \frac{1}{2}m_0^2|\phi|^2 \tag{2}$$

From this Lagrangian the field equation is derived (Klein-Gordon equation), and this equation is the same as the wave equation for one single particle. The mean value of the wave equation is in turn equivalent to the classical single particle equation of motion. Thus the terms in L which are quadratic in the field and do not contain any derivatives will sum up to give the mass term in the non-relativistic limit.

Now, the Lagrangian in eq. (2) contains a term $e^2\phi^*A^\mu A_\mu\phi$; this can be considered as an additional mass term for the field ϕ. To this end, one must average on the field $A_\mu(x)$; the effective mass turns out to be equal to $\Delta m^2 = e^2\langle|\mathbf{A}(x)|^2\rangle$ (see below). Note that the terms linear in A_μ vanish in the average.

2.2 Computation of $\langle|\mathbf{A}(x)|^2\rangle$

In order to make any computation with the vector potential $A_\mu(x)$ we need to fix a gauge. For a pure radiation field it is convenient to choose the (non covariant) radiation gauge, in which $A_0 = 0$ and $\mathrm{div}\mathbf{A} = 0$. We expect the final physical results to be independent from the gauge, and this is indeed the case.

We are thus left with the term involving $\mathbf{A}$, and the contribution to the bare mass is the average value of the square module of $\mathbf{A}$ at any point, namely $\langle|\mathbf{A}(x)|^2\rangle$. This is a positive-definite quantity and its average is not zero in general. For a quantum field it would be an infinite quantity; it is impossible to compute the square of a quantum field at a given point, or better, if we compute in quantum field theory a correlation function like $\langle\mathbf{A}(x)\mathbf{A}(y)\rangle$ and let $x \to y$, we find a divergent quantity. This is true both in the "old fashioned" formalism with field operators and in the equivalent formalisms based on functional integration over classical fields.

As a first application let us consider the vector potential associated to a black body distribution at a given temperature. Beeing immersed in this radiation, the scalar particles described by the field ϕ will have an effective mass slightly larger than the bare mass m_0.

In principle the mass shift could be measured by applying an external field – for instance, through the cyclotron frequency. In practice, however, there are several spurious factors affecting mass measurement, whose effect is larger than the mass shift due to the black body radiation bath (for instance, the Bremsstrahlung, for small orbits).

We recall that in radiation gauge one has (in units $c = 1$)

$$\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} \tag{3}$$

The electromagnetic energy density is

$$u(\mathbf{x}) = -\frac{1}{8\pi}\left[\mathbf{E}^2(\mathbf{x}) + \mathbf{B}^2(\mathbf{x})\right] \tag{4}$$

Take a vector potential with the form of a plane wave

$$\mathbf{A} = \mathrm{Re}\mathbf{A}_0 e^{i(\mathbf{kx}-\omega t)} \tag{5}$$

with $|\mathbf{k}| = \omega$. By differentiating with respect to time one finds

$$|\mathbf{E}|^2 = |\mathbf{A}_0|^2\omega^2 \sin^2(\mathbf{kx} - \omega t) \tag{6}$$

and after averaging the sine square

$$\langle|\mathbf{A}(\mathbf{x})|^2\rangle = \frac{u_\omega(\mathbf{x})}{\omega^2} \tag{7}$$

where $u_\omega(\mathbf{x})$ is the black body spectral energy density. Inserting the Planck distribution and integrating on frequency one has

$$\langle|\mathbf{A}(\mathbf{x})|^2\rangle_{thermal} = \int d\omega \frac{1}{\omega^2}\frac{\hbar\omega^3}{e^{\hbar\omega/kT} - 1} = const.\frac{(kT)^2}{\hbar} \tag{8}$$

The mass shift is given by

$$\Delta m_{thermal} = const.\sqrt{\alpha}kT \tag{9}$$

For an electron (admitted we can approximate it with a scalar particle in this context – see also [9]) one finds at room temperature $\Delta m/m \sim 10^{-7}$, which is smaller than the nominal error on the electron mass quoted in the *Review of Particles Properties* [1].

The mass shift becomes significant for a hot plasma. If the plasma is dense, the average $\langle|\mathbf{A}(\mathbf{x})|^2\rangle$ will be itself defined in part by fluctuations and correlations in the charge density [13]. Therefore the mass shift for one kind of particles in the plasma depends in general on which other particles are present. For instance, in an electron-proton plasma we expect Δm to be larger than in a plasma containing electrons and heavy ions.

2.3 Electron mass from a ZPF cut-off?

Instead of Planck distribution, we can insert into eq. (7) the "zero-point field (ZPF) spectrum", of the form $u_\omega \sim \omega^3$ (see [2]). This spectrum is Lorentz-invariant. Both Quantum Electrodynamics (QED) and Stochastic Electrodynamics (SED) lead to this expression for the frequency spectrum of the zero-point electromagnetic radiation (see also [10] and ref.s), although the approach of the two theories is different, as is the predicted complete form of the zero-point modes.

It is clear that the integral of a ω^3-spectrum is infinite, and one needs a frequency cut-off in order to obtain a finite correction Δm, to be interpreted as an increase in the particles' inertia due to the interaction with the zero-point field.

A simple dimensional analysis shows, however, that it is not trivial to express the cut-off in terms of fundamental constants, in a meaningful and consistent way. (The same difficulty also emerges when one tries to replace the traditional "infinite renormalization" of QED with a more physical "finite renormalization" – see Section 5.) If, for instance, one writes a quantity with dimension of mass using the constants e, $\hbar$, c and a length λ (corresponding to the frequency cut-off), one finds that λ is of the order of 10^{-13} cm. It is known, however, that the electron does not exhibit any finite size of this magnitude order.

We can also introduce the gravitational constant, through the Planck mass $m_P \sim 10^{19}\ GeV/c^2 \sim 10^{-33}\ cm^{-1}$ in natural units (see Section 1), or the Planck length l_P, which is thought to be a sort of fundamental length. We can write a quantity with the dimension of length, to be interpretated *a posteriori* as a natural cut-off on the ZPF, through the ratio $m_e/m_P^2 \sim 10^{10}/10^{66}\ cm \sim 10^{-56}\ cm$.However this is smaller than l_P.

Also note that reasonable estimates should yield a consistent behavior in certain limits, for instance $l_P \to 0$ or $\lambda \to 0$.

In order to go beyond simple *Ansätze* based upon dimensional analysis, detailed microscopic models of the particle/ZPF interaction can be helpful, suggesting suitable adimensional weighing functions of ω, to be inserted into the integral. For instance, according to Haisch and Rueda [11], scattering of the zero-point electromagnetic field by the quarks and electrons constituting matter would result in an acceleration-dependent reaction force that would appear to be the origin of inertia of matter. In the subrelativistic case this inertia reaction force is exactly newtonian and in the relativistic case it exactly reproduces the well known relativistic extension of Newton's Law. In this model the interaction between the particle and the zero-point field occurs at a resonance frequency, which thus defines the mass of the particle.

3 Does the relation $\Delta m^2 = e^2\langle|\mathbf{A}(x)|^2\rangle$ make sense in QFT?

In the previous Section we found for the mass shift of a charged particle in an electromagnetic field the expression $\Delta m^2 = e^2\langle|\mathbf{A}(x)|^2\rangle$. This is meaningful for classical thermal field theory or SED, but not for QED, even though these theories are under several respects almost equivalent [2, 10]. We shall see in Section 4 that the approach of quantum field theory to the mass shift problem is different and implies a procedure of "cancellation of infinities" called mass renormalization. In order to understand the intellectual path which led to quantum field theory in its present form, we are going to recall here its main features.

3.1 The main features of QFT

Quantum Field Theory results from the application of quantum mechanics to (relativistically invariant) classical field theory. It is one of the major achievements of the physics of the 20.

century, and a theory tested experimentally with high precision, especially for electrodynamics. In QFT the classical fields are promoted to operators and satisfy suitable commutation relations, like any operator in quantum mechanics. The quantization procedure of the system resembles that of ordinary quantum mechanics, except for the fact that the fields behave like (non-commuting) distributions rather than functions. This implies that the square of a field at a given point is a divergent quantity; physical expressions involving squared fields, like the energy density, must be regularized by subtracting divergent quantities. Further divergences arise from perturbation theory, in the sums over intermediate states (”virtual particles”). A mathematical procedure has been invented, called “renormalization”, that allows to eliminate the infinities from physical expressions in an unambiguous way (see Section 4.2). Field theories which are renormalizable are regarded as good candidates for the description of elementary particles and their interactions.

Most results of QFT are obtained through perturbative expansions, even though a number of direct consequences of the basic structure of the theory exist (the so-called current algebra theorems, and others). An alternative approach to QFT, pioneered by R. Feynman and based on a formal “functional integral”, allows to re-obtain the perturbative expansion of operator-based QFT in a simpler way. The functional integration is performed over classical field configurations, weighed with the oscillating factor $e^{iS/\hbar}$, which is strongly peaked at the classical trajectory but also admits some fluctuations. In certain special cases where the functional integral can be properly defined, the complete non-perturbative equivalence between this formulation and the operator formulation has been proved: the two theories give the same transition amplitudes.

One of the most relevant applications of QFT, in addition to QED, is the standard model of electroweak and strong interactions. This model accounts for all the main features of these interactions, and several fine details and theoretical predictions, even at high orders, have been confirmed experimentally. Much of the predictive power of QFT resides in its ability to accomodate in a clear way all the symmetries of the various physical systems. Furthermore, it is often possible to write “effective quantum field theories”, which describe a given system in a certain energy range, by enclosing in suitable parameters all the physical effects concerning a different (usually larger) energy range. Famous wizards of QFT, like S. Weinberg, believe that this is indeed the true essence of QFT and that any quantum field theory should be regarded as an effective theory. In this context, even the renormalizability requirement has been somewhat relaxed in the last years [12].

3.2 The Higgs mechanism

Besides the many successes, there are still a few unsolved problems in QFT. One of these is the incompatibility of QFT with Einstein General Relativity, which is still recognized as the best available theory of gravitational phenomena. It is therefore impossible today to analyze the gravitational field at the quantum level, in a way similar to what is done for the electromagnetic field.

The second problem is the so-called cosmological constant paradox: the vacuum fluctuations predicted by QFT contain a huge Lorentz-invariant energy density, which corresponds in turn to a large cosmological term in the Einstein equations and imply an (unobserved) strong curvature of the universe.

The third problem is that the Higgs boson has not been observed yet and it seems unlikely that any clear experimental evidence of its existence will ever be found. The Higgs boson is a heavy exotic particle corresponding to a field whose existence is necessary to ensure renormalizability (and thus consistence) of the standard model of the electroweak interactions. According to QFT, the Z^0 and $W^{\pm}$ vector bosons which carry the weak interaction can become massive without spoiling the theory's renormalizability only if a Higgs field exists, which is in a state of spontaneous symmetry breaking.

As a consequence of spontaneous symmetry breaking, the vacuum expectation value of the Higgs field is constant and non-zero. This gives rise to a mass for the vector bosons *in a similar way as described in Section 2, but without leading to infinities*, since the vacuum expectation value of the Higgs field is a classical quantity unrelated to vacuum fluctuations. Therefore many theorists regard the Higgs field not as a real entity, but only as a mathematical trick ensuring the consistency of the theory – something very similar to a Lorentz-invariant aether. It is important to recall that in the standard model the Higgs field provides a mass not only for the vector bosons, but also for all fermions, through coupling constants which are adjusted as free parameters!

3.3 Discrete QFT?

Several theorists have been looking for an alternative to the Higgs mechanism, in particular some proposed models try to eliminate infinities by introducing a fundamental length in spacetime. Many different kinds of such "discrete field theory" [14] have been proposed (see for instance the review by Garay [15]). Most of them take Planck length as the fundamental length, and Planck mass as the energy cut-off. In this context the square of a field at a given point becomes a meaningful quantity.

Up to now, however, there does not exist any model which (1) is mathematically affordable and comparable for efficiency to QFT; (2) is natural – that means, independent on arbitrary choices in the discretization scheme. For some recent work on this matter, see [16].

4 Vacuum fluctuations and mass renormalization in QFT

In this Section we recall the main steps of the so-called Feynman mass renormalization procedure. It should be emphasized that the need for renormalization is rather general and is not unique to the relativistic field theories. The examples of electron mass renormalization in a lattice and in a plasma display this very clearly. For the relativistic theory, the situation is the same except for two important distinctions. First, the renormalization contributions are generally infinite. Second, there appears to be no way to switch off the interaction, and the bare quantities are not measurable.

Let us first recall the relationship between the Feynman propagator and the physical mass of a particle.

4.1 Premise: Feynman propagator vs. mass

In elementary quantum mechanics one often considers the "propagator" $G(t_2 - t_1)$ which takes a state ψ forward in time:

$$\begin{aligned} G(t_2 - t_1)|\psi(t_1)\rangle &= |\psi(t_2)\rangle \quad if \quad t_2 > t_1 \\ G(t_2 - t_1)|\psi(t_1)\rangle &= 0 \quad if \quad t_2 < t_1 \end{aligned} \tag{10}$$

The propagator can also be expressed as a formal Fourier transform:

$$G(t) = \int_{-\infty}^{+\infty} \frac{ds}{2\pi} e^{-ist} \tilde{G}(s) \tag{11}$$

where

$$\tilde{G}(s) = \frac{i}{s - H + i\varepsilon} \tag{12}$$

and H is the Hamiltonian of the system. If we take a free electron with momentum $\mathbf{p}$, we have

$$H|\mathbf{p}\rangle = E_p|\mathbf{p}\rangle \tag{13}$$

where $E_p = (\mathbf{p}^2 + m^2)^{1/2}$.

Therefore the amplitude to find the electron in the same state at time $t > 0$ is

$$\begin{aligned} Amp(t) &= \int_{-\infty}^{+\infty} \frac{ds}{2\pi} e^{-ist} \langle \mathbf{p}|G(s)|\mathbf{p}\rangle \\ &= \int_{-\infty}^{+\infty} \frac{ds}{2\pi} e^{-ist} \frac{i}{s - E_p + i\varepsilon} \end{aligned} \tag{14}$$

This integral can be performed exactly, and the result is $Amp(t) = \exp(-iE_p t)$ as expected. More generally we can say that the time rate of change of the phase of $Amp(t)$, i.e. the location of the pole in $\langle \mathbf{p}|G(s)|\mathbf{p}\rangle$, measures the electron energy (which, for given $\mathbf{p}$, defines the inertial mass). In the presence of vacuum fluctuations, the position of the pole is shifted, as we show in the next Section.

4.2 Self-energy for a scalar field

Let us first consider again, for simplicity, the quantum field theory of a scalar field ϕ (this time self-interacting); let the lagrangian density be separated into free and interacting parts

$$L = L_0 + L_1 \tag{15}$$

with

$$L_0 = \frac{1}{2}\left[\partial_\alpha \phi \partial^\alpha \phi - m_0^2 \phi^2\right] \qquad \text{and} \qquad L_1 = -\frac{1}{4!}\phi^4 \tag{16}$$

The Feynman propagator in momentum space, or "2-point Green function" $\Delta(p)$, is the Fourier transform of the vacuum expectation value of the time-ordered product of two fields (which gives the probability amplitude for a particle to be annihilated in x and created again in y):

$$\Delta(p) = \int d^4x e^{-ip(x-y)} \langle 0|T(\phi(x)\phi(y))|0\rangle \tag{17}$$

Diagrammatically, the complete propagator can be obtained as an infinite sum over 1-particle irreducible self-energy insertions (see Fig. 1). Each line of the diagram represents a propagator of the non-interacting field with lagrangian L_0, given by

$$\Delta_0(p) = \frac{1}{p^2 - m_0^2 + i\varepsilon} \tag{18}$$

where m_0 is the "bare" mass, the mass of the non-interacting particle (clearly not observable). The symbol $\Sigma(p^2)$ represents the sum of all possible connected vacuum polarization diagrams, for instance those in Fig. 2.

The case of QED is more complex than that of a self-interacting scalar field, because we have two different interacting fields. The correction analogous to $\Sigma(p^2)$ above is the so-called electron self-energy, with graphs as in Fig. 3. We could describe these diagrams by saying that the electron emits a photon and then reabsorbs it (B); to higher order, the photon may itself split into two electrons which then reannihilate (C), etc. Even for the emission of a single photon, the probability of the process is given by a sum over all possible 4-momenta p of the photon. The latter is a "virtual" particle because it does not satisfy the energy-momentum relation $p^2 = 0$ typical of a free photon; it is not able to emerge from the process as a real particle and the corresponding electromagnetic field is not regarded as a wave, but as a vacuum fluctuation limited in space and time. Self-energy diagrams are proportional to powers of the Planck constant $\hbar$ and represent quantum processes; the lifetime of virtual photons is so short – and consequently the indetermination of their energy so large – that they can carry any 4-momentum p.

It is quite clea, intuitively, that these processes of emission/re-absorption of virtual particles oppose to acceleration and therefore increase the inertial mass of the particles. Below we shall quote the final results of the Δm computation for QED and also for scalar QED. Here let us continue with the explicit example of the self-interacting scalar field, which is much simpler.

The sum $\Delta(p)$ is a geometric series and therefore has the remarkable property that $\Sigma(p^2)$ contributes to the free propagator only as an addition to the squared bare mass m_0:

$$\Delta(p) = \frac{1}{p^2 - m_0^2 - \Sigma(p^2) + i\varepsilon} \tag{19}$$

To one-loop order one finds for the contribution to the squared bare mass (diagram A of Fig. 2)

$$\Sigma(p^2) = \frac{\lambda}{2} \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 - m_0^2 + i\varepsilon} \tag{20}$$

This quantity is divergent, because there is no upper limit for the p^2 of virtual particles. However, by differentiating it with respect to the external momentum one eventually obtains convergent quantities. Thus $\Sigma(p^2)$ is Taylor-expanded from an arbitrary point m^2, obtaining

$$\Sigma(p^2) = \Sigma(m^2) + (p^2 - m^2)\Sigma'(m^2) + \Sigma_{finite}(p^2) \tag{21}$$

with $\Sigma_{finite}(m^2) = 0$ and $\Sigma'_{finite}(m^2) = 0$.

As we have seen, the physical mass is defined in QFT as the position of the pole of the propagator. Since up to this point m^2 is arbitrary, we can choose it to satisfy the equation

$$m_0^2 + \Sigma(m^2) = m^2 \tag{22}$$

Thus we find for $\Delta(p)$

$$\Delta(p) = \frac{1}{p^2 - m_0^2 - \Sigma(m^2) - (p^2 - m^2)\Sigma'(m^2)\Sigma_{finite}(p^2) + i\varepsilon} \tag{23}$$

One can see that this expression has a pole at $p^2 = m^2$. Thus m is the physical mass and is related to the bare mass through eq. (22). This is called the "mass renormalization". Since $\Sigma(m^2)$ is divergent, the bare mass m_0^2 must also be divergent so that the physical mass m^2 is finite. The divergent term $\Sigma'(m^2)$ can be removed by rescaling the field operator and finally one has

$$\Delta(p) = \frac{1}{p^2 - m^2 - \Sigma_{finite}(p^2) + i\varepsilon} \tag{24}$$

The finite part $\Sigma_{finite}(p^2)$ does not affect the physical mass, but it does affect diagrams with internal electron propagators, like that of "Compton" scattering (Fig. 4). Here the virtual electron (a) propagates according to the full $\Delta(p)$, including the part $\Sigma_{finite}(p^2)$ which accounts for the vacuum fluctuations. Such are, in the QFT view, the only observable effects of vacuum fluctuations on inertia (confirmed by the experiments). These effects only involve *virtual* particles! They amount to a slight dependence of their effective mass on the exernal momentum exchange.

But the point is: does the convention (22) make any physical sense? It is true that the "mass of a non-interacting particle" is something undefined, but why take it to be infinite? If we take it to be zero, then we can suppose to cut-off the integral on momenta in (20), obtaining a finite value for $\Sigma(m^2)$. In this way, instead of ending up with a parameter m which is finite by definition but otherwise arbitrary, we could solve the equation

$$\Sigma(m^2) = m^2 \tag{25}$$

and find m^2 as a function of the physical cut-off (because Σ contains it).

We shall see in the next Section that the feasibility of this idea depends crucially on the form of the function Σ.

5 The "bare mass": infinite, finite or zero?

In the previous Section we derived the mass renormalization condition (22) and we wondered whether one could consistently set $m_0 = 0$ (zero bare mass).

Let us first try this explicitly for the $\lambda\phi^4$ theory, then for the so-called "scalar QED" and finally for the QED with spinors. After analytical continuation, the integral (20) can be computed in "Euclidean space", i.e. with $k^2 = k_0^2 + \mathbf{k}^2$ and one finds, apart from a numerical factor coming from the integration over 4D angles

$$\Sigma \sim \int_0^\infty du \frac{u^3}{u^2 - m_0^2} \tag{26}$$

where $u = \sqrt{k^2}$. As can be seen already in eq. (20), Σ does not depend on the external momentum p in this special case. After we cut the integration over the virtual momenta with a cut-off M, Σ depends only on M and on the bare mass m_0. We then have

$$\Sigma \sim \int_0^M du \frac{u^3}{u^2 - m_0^2} = \frac{M^2}{2} + \frac{m_0^2}{2} \ln(M^2 - m_0^2) \tag{27}$$

Eq. (22) for the renormalized mass m becomes simply

$$m = M \tag{28}$$

i.e., the renormalized mass equals the cut-off mass scale.

What is the cut-off mass M? It represents any quantity which can reasonably constitute an upper bound on the energy-momentum of the virtual particles of the field. It is believed that, in general, M cannot exceed M_{Planck}, but the physical cut-off could be smaller, for some reason, in specific cases.

A similar result is found for "scalar QED", the quantum field theory of the electromagnetic field interacting with massive bosons described by a scalar field ϕ. In addition to the interaction vertex in fermion QED, here we have an interaction vertex with two ϕ-particles and two photons

(Fig. 5). This gives a self-energy (vacuum mass) diagram similar to that of the self-interacting $\lambda\phi^4$ theory (Fig. 2.A), and Σ is given by

$$\Sigma = \frac{e^2}{\pi} \int d^4k \frac{1}{k^2 + i\varepsilon} \tag{29}$$

This is a special case of the previous integral; also in this case Σ does not depend on the external 4-momentum p; moreover, it does not depend on the bare mass m_0 of the scalar field. Inserting a cut-off M we find, as before, $m = M$.

In QED with spinors the renormalization procedure is much more complicated, but it follows similar lines (see for instance [17]). The mass enters the propagator linearly and not quadratically. The divergent part of the self-energy Σ depends only logarithmically on the cut-off. The difference between renormalized mass and bare mass is given by

$$m - m_0 = \left[\Sigma(p^2, M)\right]_{\sqrt{p^2}=m} = m_0 \frac{3\alpha}{4\pi} \left(\ln \frac{M^2}{m_0^2} + \frac{1}{2}\right) \tag{30}$$

As usual, the divergence in Σ is re-absorbed by subtraction from an infinite bare mass. What if we try to set $m_0 = 0$ and find m as a function of M? We see from (30) that this is impossible, or better setting $m_0 = 0$ we find $m = 0$, too. If, however, we admit that the bare mass is different from zero (and not much smaller than the observed mass m) we can conclude that the percentual correction due to vacuum effects, namely

$$\frac{m - m_0}{m_0} = \frac{3\alpha}{4\pi} \left(\ln \frac{M^2}{m_0^2} + \frac{1}{2}\right) \tag{31}$$

is reasonable even if M equals the maximum conceivable cut-off, the Planck mass. This is due to the mild logarithmical dependence and to the smallness of α.

We conclude that the prescription $M \to \infty$ is, to some extent, a matter of taste. On one hand it avoids problems and ambiguities related to the choice of the cut-off. On the other hand, it somehow "sweeps the problem under the rug".

The idea that the bare mass could be zero and the real mass entirely due to vacuum fluctuations has a philosophical and epistemological appeal, since it allows to regard the mass not as a primitive concept, but as the consequence of the interaction of the particle with a field – an already known entity. This does not hold, of course, for finite renormalization, which still implies a given non-zero bare mass.

From the practical point of view, however, the particle mass remains a very economical parameter, more than any parameters describing the detailed particle/ZPF interaction.

6 Casimir mass correction for conducting macroscopic bodies

After discussing in Sect.s 4 and 5 the mass renormalization procedure for a pointlike particle, we recall here a less known prediction of quantum field theory: The mass of any conducting macroscopic body is affected by electromagnetic vacuum fluctuations. This kind of mass modification is usually very small and is related to the Casimir effect [18].

The Casimir effect is the most clear manifestation of the physical reality of the electromagnetic zero-point field. It generates tiny forces between uncharged conducting bodies, which have been experimentally detected. According to quantum field theory, these forces arise because any conducting body enforces a boundary condition on the electromagnetic field (the field must vanish inside it), and therefore modifies the frequency spectrum of the zero-point field modes. The different between the total zero-point energy in the presence of the body and the zero-point energy of empty space is called the Casimir energy E_C of the body. If there are more bodies, this energy depends on their distance and its first derivative gives the Casimir force F_C.

For realistic conductors, which exclude the field only up to a certain cut-off frequency, E_C depends on the cut-off, but F_C usually does not. Without the cut-off, E_C would actually be infinite, since the Casimir energy *density* goes as r^{-4} near the surface of a perfect conductor [19].

Therefore the correction to the conductor's inertial mass, $\Delta m = E_C/c^2$, is infinite for perfect conductors, while for real conductors it is a (yet unknown) function of their size, geometrical features, and cut-off frequency – typically the plasma frequency.

The mass correction can be computed exactly for a dielectric ball (at least in some special cases, like for instance when $\varepsilon \neq 1$ and $\mu \neq 1$ but $\varepsilon\mu = 1$). In this case the presence of the body changes the total electromagnetic zero-point energy by a finite amount. The total zero-point field energy E_C can be expressed as a power series in the parameter $\xi = (1-\mu)/(1+\mu)$ [20]. The leading term is equal to ξ^2/a (a radius of the ball), with a proportionality constant of order 1. For $a \sim 1$ *cm* or larger, this energy is very small – a fraction of eV – but it would be significant for a hypothetical dielectric ball of atomic size. The case of perfect conductivity is recovered when $\xi \to 1$, in which case, however, the series diverges as expected.

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